The effect of modified gravity on weak lensing

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We study the effect of modified gravity on weak lensing in a class of scalar-tensor theory that includes $f(R)$ gravity as a special case. These models are designed to satisfy local gravity constraints by having a large scalar-field mass in a region of high curvature. Matter density perturbations in these models are enhanced at small redshifts because of the presence of a coupling Q that characterizes the strength between dark energy and non-relativistic matter. We compute a convergence power spectrum of weak lensing numerically and show that the spectral index and the amplitude of the spectrum in the linear regime can be significantly modified compared to the ΛCDM model for large values of $|Q|$ of the order of unity. Thus weak lensing provides a powerful tool to constrain such large coupling scalar-tensor models including $f(R)$ gravity.

I. INTRODUCTION

The observations of the Supernovae Ia (SN Ia) in 1998 [1] opened up a new research paradigm known as Dark Energy (DE). In spite of the tremendous effort over the past ten years, we have not yet identified the origin of DE responsible for the late-time accelerated expansion. Many DE models have been proposed so far to alleviate the theoretical problem of the cosmological constant scenario [2, 3]. We can broadly classify these models into two classes: (i) "changing gravity" models and (ii) "changing matter" models. The first class includes $f(R)$ gravity [4], scalar-tensor models [5] and braneworld models [6], whereas scalar-field models such as quintessence [7] and k-essence [8] are categorized in the second class.

While changing matter models lead to dynamical evolution for the equation of state of DE, it is not easy to distinguish them from the cosmological constant scenario in current observations. Meanwhile, if we change gravity from General Relativity, the models need to pass local gravity tests as well as cosmological constraints. In this sense it is possible to place stringent experimental and observational constraints on changing gravity models.

In fact there have been a burst of activities to search for viable modified gravity DE models. In the so-called $f(R)$ gravity where f is a function of the Ricci scalar R, it was found that the model $f(R) = R - \alpha/R^n$ $(\alpha, n > 0)$ proposed in Refs. [4] is unable to satisfy the stability condition $(f_{,RR} \equiv \mathrm{d}^2 f/\mathrm{d} R^2 > 0)$ for perturbations [9], cosmological viability [10] and local gravity constraints (LGC) [11]. Recently a number of authors proposed viable $f(R)$ DE models that satisfy all these requirements [12, 13, 14, 15, 16, 17, 18, 19, 20]. For example, the model

$f(R) = R - \alpha R^n$ with $\alpha > 0, 0 < n < 1$ is consistent with LGC for $n < 10^{-10}$ [21] while at the same time satisfying stability and cosmological constraints. However it is difficult to distinguish this model from the ΛCDM cosmology because of the tight bound on the power n coming from LGC.

The $f(R)$ models proposed by Hu and Sawicki [15] and Starobinsky [16] are designed to satisfy LGC in the region of high density where local gravity experiments are carried out. Moreover it is possible to find an appreciable deviation from the ΛCDM model as the Universe evolves from the matter-dominated epoch to the late-time accelerated era. In fact the equation of state of DE in these models exhibits peculiar evolution at small redshifts [14, 18]. In addition, for the redshift smaller than a critical value z_k, the growth rate of matter density perturbations is larger than in the case of General Relativity [16, 18].

Recently the analysis in $f(R)$ gravity was extended to a class of scalar-tensor DE models, i.e., Brans-Dicke theory with a scalar field potential $V(\phi)$ [22]. By introducing a constant Q with the relation $1/(2Q^2) = 3 + 2\omega_{\mathrm{BD}}$ (ω_{BD} is a Brans-Dicke parameter), one can reduce this theory to the one given by the action (2). The constant Q characterizes the coupling between dark energy and non-relativistic matter. If the scalar field ϕ is nearly massless, the coupling is constrained to be $|Q| \lesssim 10^{-3}$ from solar system experiments [22]. However, if the field ϕ is massive in the region of high density, it is possible to satisfy LGC even when $|Q|$ is of the order of unity. In fact, in the context of $f(R)$ gravity ($Q = -1/\sqrt{6}$), the models of Hu and Sawicki [15] and Starobinsky [15] are designed in such a way that the field is sufficiently massive in the regime $R \gg R_0$ (R_0 is the present cosmological Ricci scalar) and that the mass becomes lighter as R approaches R_0. For general coupling Q, the potential given in Eq. (5) can be compatible with both local gravity and cosmological constraints.

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The scalar-tensor models mentioned above show deviations from the ΛCDM model at late times and hence they can leave a number of interesting observational signatures. In Ref. [22] several bounds on the coupling Q and model parameters were derived by considering the evolution of matter density perturbations as well as LGC. It was found that there exists allowed parameter space of model parameters even when $|Q|$ is of the order of unity.

In this paper we shall study the effect of such modified gravity models on weak lensing observations [23]. Since weak lensing carries the information of perturbations at low redshifts, it is expected that this sheds light on revealing the nature of DE [24, 25, 26, 27, 28, 29, 30, 31, 32]. In Refs. [33, 34] a convergence power spectrum of weak lensing was derived in scalar-tensor theories with the Lagrangian density $\mathcal{L} = F(\phi)R/2 - (\nabla\phi)^2/2 - V(\phi)$. In these theories a deflecting lensing potential $\Phi_{\rm wl}$ is modified compared to General Relativity due to the different evolution of gravitational potentials. This gives rise to the change of the convergence power spectrum, which provides a powerful tool to distinguish modified gravity from the ΛCDM model.

The lensing potential $\Phi_{\rm wl}$ is sourced by matter density perturbations. The equation for matter perturbations in scalar-tensor models was derived in Ref. [35] under the approximation on sub-horizon scales (see also Ref. [36]). This analysis can be generalized to the theories with the Lagrangian density $f(R, \phi, X)$ (where $X = -(\nabla\phi)^2/2$), in which $\Phi_{\rm wl}$ was obtained analytically [37]. The DGP braneworld model also leads to the modification to the lensing potential [38]. Thus the effect of modified gravity generally manifests itself in weak lensing observations.

In this work we focus on scalar-tensor models (2) with a large coupling Q and evaluate the convergence power spectrum to find signatures of the modification of gravity in weak lensing. This analysis is general in the sense that $f(R)$ gravity is included as a special case. In Sec. II we review our scalar-tensor models and present cosmological background equations to find dark energy dynamics. In Sec. III we derive the form of the convergence power spectrum as well as the equation for the deflecting potential $\Phi_{\rm wl}$. In Sec. IV we compute the convergence spectrum numerically and estimate the effect of modified gravity on weak lensing. We conclude in Sec. V.

II. MODIFIED GRAVITY MODELS

We start with the following action

$$S = \int {\rm d}^4x\sqrt{-g}\left[\frac{1}{2}\chi R - \frac{\omega_{\rm BD}}{2\chi}(\nabla\chi)^2 - V(\chi)\right] + S_m(g_{\mu\nu}, \Psi_m),\tag{1}$$

where χ is a scalar field coupled to the Ricci scalar R, $\omega_{\rm BD}$ is a constant parameter, $V(\chi)$ is a field potential, and S_m is a matter action that depends on the metric $g_{\mu\nu}$ and matter fields Ψ_m. The action (1) corresponds

to Brans-Dicke theory [39] with a potential $V(\chi)$. In the following we use the unit $8\pi G = 1$, but we restore the bare gravitational constant G when it is required.

Setting $\chi = F = e^{-2Q\phi}$, where Q is a constant and $\phi = -1/(2Q)\ln\chi$ is a new scalar field, we find that the action (1) is equivalent to

$$S = \int {\rm d}^4x\sqrt{-g}\left[\frac{1}{2}FR - \frac{1}{2}(1 - 6Q^2)F(\nabla\phi)^2 - V\right] + S_m(g_{\mu\nu}, \Psi_m),\tag{2}$$

where Q is related with the Brans-Dicke parameter $\omega_{\rm BD}$ via the relation $1/(2Q^2) = 3 + 2\omega_{\rm BD}$ [22]. The $f(R)$ gravity corresponds to the coupling $Q = -1/\sqrt{6}$, i.e., $\omega_{\rm BD} = 0$ [40].

In the absence of the potential V the Brans-Dicke parameter is constrained to be $\omega_{\rm BD} > 4.0 \times 10^4$ from solar system experiments [41], which gives the bound $|Q| < 2.5 \times 10^{-3}$. If the potential V is present, it is possible to satisfy solar system constraints even when $|Q|$ is of the order of unity by having a large mass in a high-curvature region. In the context of $f(R)$ gravity, the following model is designed to satisfy LGC [18]:

$$f(R) = R - \mu R_c[1 - (R/R_c)^{-2n}],\tag{3}$$

where μ, R_c, n are positive constants, and R_c is roughly of the order of the present cosmological Ricci scalar R_0. Note that this satisfies the stability condition $f_{,RR} > 0$ for $R \geq R_1$ (R_1 is a Ricci scalar at a late-time de-Sitter point) unlike the model $f(R) = R - \alpha/R^n$ ($\alpha, n > 0$) [16, 18]. In the limit $R \gg R_c$ the above model approaches the ΛCDM model, which allows a possibility to be consistent with LGC in the region of high density.

In fact, the model (3) satisfies LGC for $n > 0.9$ [21] through a chameleon mechanism [42] because of the presence of an effecive potential $V = (RF - f)/2$ with the dynamical field $\phi = (\sqrt{6}/2)\ln F$. The field potential in this case is given by

$$V(\phi) = \frac{\mu R_c}{2}\left[1 - \frac{2n+1}{(2n\mu)^{2n/(2n+1)}}(1 - e^{2\phi/\sqrt{6}})^{\frac{2n}{2n+1}}\right].\tag{4}$$

The models proposed by Hu and Sawicki [15] and by Starobinsky [16] reduce to this form of the potential in the high-curvature region ($R \gg R_c$) where local gravity experiments are carried out. When $R \gg R_c$ the field ϕ is almost frozen at instantaneous minima around $\phi = 0$ characterized by the condition $e^{2\phi/\sqrt{6}} = 1 - 2n\mu(R/R_c)^{-(2n+1)}$ with a large mass squared $M^2 \equiv V_{,\phi\phi} \propto \phi^{-\frac{2n+2}{2n+1}}$. These minima are sustained by an effective coupling Q between non-relativistic matter and the field ϕ [21].

For arbitrary coupling Q with the action (2), one can also construct viable models by generalizing the analysis of $f(R)$ gravity. An explicit example of the potential consistent with LGC is given by [22]

$$V(\phi) = V_1\left[1 - C(1 - e^{-2Q\phi})^p\right],\tag{5}$$

where $V_1 > 0$, $C > 0$, $0 < p < 1$. This is motivated by the potential (4), which means that the $f(R)$ model (3) is recovered by setting $p = 2n/(2n+1)$. The analysis using the potential (5) with the action (2) is sufficiently general to understand essential features of modified gravity models that satisfy local gravity and cosmological constraints. As p gets closer to 1, the field mass in the region of high-curvature tends to be heavier so that the models are consistent with LGC. In Ref. [22] it was found that the constraints coming from solar system tests and the violation of equivalence principle give the bounds $p > 1 - 5/(9.6 - \ln_{10}|Q|)$ and $p > 1 - 5/(13.8 - \ln_{10}|Q|)$, respectively. In $f(R)$ gravity with the potential (4) these bounds translate into the conditions $n > 0.5$ and $n > 0.9$, respectively [21].

Let us review cosmological dynamics for the action (2) with the potential (5) in the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$, where t is cosmic time and $a(t)$ is the scale factor. As a source term for the matter action S_m, we take into account a non-relativistic fluid with energy density ρ_m and a radiation with energy density ρ_{rad}. These obey the usual conservation equations $\dot{\rho}_m + 3H\rho_m = 0$ and $\dot{\rho}_{rad} + 4H\rho_{rad} = 0$, where $H \equiv \dot{a}/a$. The variation of the action (2) leads to the following equations of motion:

$$3FH^2 = \frac{1}{2}(1 - 6Q^2)F\dot{\phi}^2 + V - 3H\dot{F} + \rho_m + \rho_{rad}, \quad (6)$$

$$2F\dot{H} = -(1 - 6Q^2)F\dot{\phi}^2 - \ddot{F} + H\dot{F} - \rho_m - \frac{4}{3}\rho_{rad}, \quad (7)$$

$$(1 - 6Q^2)F\left(\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{F}}{2F}\dot{\phi}\right) + V_{,\phi} + QFR = 0, \quad (8)$$

where $R = 6(2H^2 + \dot{H})$.

In order to solve the background equations (6)-(8) numerically, we introduce the dimensionless variables

$$x_1 = \frac{\dot{\phi}}{\sqrt{6}H}, \quad x_2 = \frac{1}{H}\sqrt{\frac{V}{3F}}, \quad x_3 = \frac{1}{H}\sqrt{\frac{\rho_{rad}}{3F}}. \quad (9)$$

We also define

$$\Omega_{DE} \equiv (1 - 6Q^2)x_1^2 + x_2^2 + 2\sqrt{6}Qx_1, \quad (10)$$
$$\Omega_{rad} \equiv x_3^2, \quad (11)$$
$$\Omega_m \equiv 1 - (1 - 6Q^2)x_1^2 - x_2^2 - 2\sqrt{6}Qx_1 - x_3^2, \quad (12)$$

which satisfy the relation $\Omega_{DE} + \Omega_{rad} + \Omega_m = 1$. Using Eqs. (7) and (8) we find

$$\frac{\dot{H}}{H^2} = -\frac{1 - 6Q^2}{2}\left[3 + 3x_1^2 - 3x_2^2 + x_3^2 - 6Q^2x_1^2 + 2\sqrt{6}Qx_1\right] + 3Q(\lambda x_2^2 - 4Q), \quad (13)$$

where $\lambda = -V_{,\phi}/V$. For the potential (5) we have

$$\lambda = \frac{2C\,p\,Qe^{-2Q\phi}(1 - e^{-2Q\phi})^{p-1}}{1 - C(1 - e^{-2Q\phi})^p}. \quad (14)$$

The effective equation of the system is defined by

$$w_{eff} \equiv -1 - 2\dot{H}/(3H^2). \quad (15)$$

Using Eqs. (6)-(8), we obtain the following equations

$$\frac{dx_1}{dN} = \frac{\sqrt{6}}{2}(\lambda x_2^2 - \sqrt{6}x_1) + \frac{\sqrt{6}Q}{2}\left[(5 - 6Q^2)x_1^2\right.$$
$$\left. + 2\sqrt{6}Qx_1 - 3x_2^2 + x_3^2 - 1\right] - x_1\frac{\dot{H}}{H^2}, \quad (16)$$

$$\frac{dx_2}{dN} = \frac{\sqrt{6}}{2}(2Q - \lambda)x_1x_2 - x_2\frac{\dot{H}}{H^2}, \quad (17)$$

$$\frac{dx_3}{dN} = \sqrt{6}Qx_1x_3 - 2x_3 - x_3\frac{\dot{H}}{H^2}, \quad (18)$$

where $N \equiv \ln(a)$ is the number of e-foldings. We note that the variable F satisfies the equation of motion: $dF/dN = -2\sqrt{6}Qx_1F$.

There exists a radiation fixed point: $(x_1, x_2, x_3) = (0, 0, 1)$ for this system. During radiation and matter eras, the field ϕ is stuck around the "instantaneous" minima characterized by the condition $V_{,\phi} + QFR = 0$, i.e.,

$$2Q\phi_m \simeq \left(\frac{2V_1pC}{\rho_m}\right)^{\frac{1}{1-p}} \ll 1, \quad (19)$$

where we used the fact that V_1 is of the order of the squared of the present Hubble parameter H_0 so that the potential (5) is responsible for the accelerated expansion today. Note that we have $F = e^{-2Q\phi_m} \simeq 1$ under the condition (19). In this region the quantity $|\lambda|$ defined in Eq. (14) is much larger than unity. The field value $|\phi_m|$ increases as the system enters the epoch of an accelerated expansion, which leads to the decrease of $|\lambda|$. The matter-dominated epoch is realized by the instantaneous fixed point characterized by $(x_1, x_2, x_3) = (\sqrt{6}/(2\lambda), [(3+2Q\lambda-6Q^2)/2\lambda^2]^{1/2}, 0)$ with $\Omega_m = 1 - (3 - 12Q^2 + 7Q\lambda)/\lambda^2 \simeq 1$ and $w_{eff} = -2Q/\lambda \simeq 0$ (because $|\lambda| \gg 1$ in this regime). In the presence of the coupling Q there exists a de-Sitter point characterized by $(x_1, x_2, x_3) = (0, 1, 0)$, $\Omega_m = 0$ and $w_{eff} = -1$, which corresponds to $\lambda = 4Q$. This solution is stable for $d\lambda/d\phi < 0$ [22] and hence can be used for the late-time accelerated expansion. See Ref. [22] for detailed analysis about the background cosmological evolution.

The mass squared, $M^2 = V_{,\phi\phi}$, is given by

$$M^2 = 4V_1CpQ^2(1 - pe^{-2Q\phi})(1 - e^{-2Q\phi})^{p-2}e^{-2Q\phi}. \quad (20)$$

Plugging the field value ϕ_m into Eq. (20), we find

$$M^2 \simeq \frac{1-p}{(2^ppC)^{1/(1-p)}}Q^2\left(\frac{\rho_m}{V_1}\right)^{\frac{2-p}{1-p}}V_1. \quad (21)$$

Since the energy density ρ_m is much larger than V_1 during the radiation and matter eras, we have that $M^2 \gg V_1 \sim H_0^2$. The mass squared M^2 decreases to the order of V_1 after the system enters the accelerated epoch. This evolution of the field mass leads to an interesting observational signature in weak lensing observations, as we will see in subsequent sections.

III. WEAK LENSING

Let us consider a perturbed metric about the flat FLRW background in the longitudinal gauge:

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j , \quad (22)$$

where scalar metric perturbations Φ and Ψ do not coincide with each other in the absence of an anisotropic stress. Matter density perturbations δ_m in the pressureless matter contribute to the source term for the gravitational potentials Φ and Ψ. The equation of δ_m for the action (2) was derived in Ref. [22] under an approximation on sub-horizon scales [3, 35, 37]. Provided that the oscillating mode of the field perturbation $\delta\phi$ does not dominate over the matter-induced mode at the initial stage of the matter era, we obtain the following approximate equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\rho_m \delta_m \simeq 0 , \quad (23)$$

where the effective gravitational "constant" is given by

$$G_{\rm eff} = \frac{1}{8\pi F} \frac{(k^2/a^2)(1 + 2Q^2)F + M^2}{(k^2/a^2)F + M^2} . \quad (24)$$

Here k is a comoving wavenumber and M^2 is given in Eq. (20) for the potential (5). Using the derivative with respect to N, Eq. (23) can be written as

$$\frac{d^2\delta_m}{dN^2} + \left(\frac{1}{2} - \frac{3}{2}w_{\rm eff}\right)\frac{d\delta_m}{dN}$$
$$- \frac{3}{2}\Omega_m \frac{(k^2/a^2)(1 + 2Q^2)F + M^2}{(k^2/a^2)F + M^2}\delta_m \simeq 0 . \quad (25)$$

The gravitational potentials Φ and Ψ satisfy

$$\frac{k^2}{a^2}\Phi \simeq -\frac{\rho_m}{2F} \frac{(k^2/a^2)(1 + 2Q^2)F + M^2}{(k^2/a^2)F + M^2}\delta_m , \quad (26)$$

$$\frac{k^2}{a^2}\Psi \simeq -\frac{\rho_m}{2F} \frac{(k^2/a^2)(1 - 2Q^2)F + M^2}{(k^2/a^2)F + M^2}\delta_m . \quad (27)$$

In order to confront our model with weak lensing observations, we define the so-called deflecting potential [34]

$$\Phi_{\rm wl} \equiv \Phi + \Psi , \quad (28)$$

together with the effective density field

$$\delta_{\rm eff} \equiv -\frac{a}{3H_0^2\Omega_{m,0}}k^2\,\Phi_{\rm wl} , \quad (29)$$

where the subscript "0" represents the present values and we set $a_0 = 1$. Using the relation

$$\rho_m = 3F_0 H_0^2\Omega_{m,0}/a^3 , \quad (30)$$

together with Eqs. (26) and (27), we get

$$\Phi_{\rm wl} = -\frac{a^2}{k^2}\frac{\rho_m}{F}\delta_m , \quad \delta_{\rm eff} = \frac{F_0}{F}\delta_m . \quad (31)$$

We write the angular position of a source to be $\vec{\theta}_S$ and the direction of weak lensing observation to be $\vec{\theta}_I$. The deformation of the shape of galaxies is characterized by the amplification matrix $\mathcal{A} = d\vec{\theta}_S/d\vec{\theta}_I$. The components of $\mathcal{A}$ are given by [23, 34]

$$\mathcal{A}_{\mu\nu} = I_{\mu\nu} - \int_0^\chi \frac{\chi'(\chi - \chi')}{\chi}\partial_{\mu\nu}\Phi_{\rm wl}[\chi'\vec{\theta}, \chi']d\chi' , \quad (32)$$

where χ is the comoving radial distance satisfying the relation $d\chi = -dt/a(t)$ along the geodesic. In terms of the redshift defined by $z = 1/a - 1$, we have that

$$\chi(z) = \int_0^z \frac{dz'}{H(z')} . \quad (33)$$

The convergence κ and the shear $\vec{\gamma} = (\gamma_1, \gamma_2)$ can be derived from the components of the 2×2 matrix $\mathcal{A}$, as

$$\kappa = 1 - \frac{1}{2}{\rm Tr}\,\mathcal{A} , \quad \vec{\gamma} = ([\mathcal{A}_{22} - \mathcal{A}_{11}]/2, \mathcal{A}_{12}) . \quad (34)$$

If we consider a redshift distribution $p(\chi)d\chi$ of the source, the convergence is given by $\kappa(\vec{\theta}) = \int p(\chi)\kappa(\vec{\theta}, \chi)d\chi$. Using Eqs. (29), (32) and (34) we obtain

$$\kappa(\vec{\theta}) = \frac{3}{2}H_0^2\Omega_{m,0}\int_0^{\chi_H} g(\chi)\chi\frac{\delta_{\rm eff}[\chi\vec{\theta}, \chi]}{a}d\chi , \quad (35)$$

where χ_H is the maximum distance to the source and

$$g(\chi) \equiv \int_\chi^{\chi_H} p(\chi')\frac{\chi' - \chi}{\chi'}d\chi' . \quad (36)$$

Since the convergence is a function on the 2-sphere it can be expanded in the form $\kappa(\vec{\theta}) = \int \hat{\kappa}(\vec{\ell})e^{i\vec{\ell}\cdot\vec{\theta}}\frac{d^2\vec{\ell}}{2\pi}$, where $\vec{\ell} = (\ell_1, \ell_2)$ with ℓ_1 and ℓ_2 integers. Defining the power spectrum of the shear to be $\langle\hat{\kappa}(\vec{\ell})\hat{\kappa}^*(\vec{\ell'})\rangle = P_\kappa(\vec{\ell})\delta^{(2)}(\vec{\ell} - \vec{\ell'})$, one can show that the convergence has a same power spectrum as P_κ [23]. It is given by [34]

$$P_\kappa(\ell) = \frac{9H_0^4\Omega_{m,0}^2}{4}\int_0^{\chi_H}\left[\frac{g(\chi)}{a(\chi)}\right]^2 P_{\delta_{\rm eff}}\left[\frac{\ell}{\chi}, \chi\right]d\chi . \quad (37)$$

In our scalar-tensor theory we have $P_{\delta_{\rm eff}} = (F_0/F)^2 P_{\delta_m}$ from Eq. (31), where P_{δ_m} is the matter power spectrum. In the following we assume that the sources are located at the distance χ_s (corresponding to the redshift z_s), which then gives $p(\chi) = \delta(\chi - \chi_s)$ and $g(\chi) = (\chi_s - \chi)/\chi_s$. This leads to the following convergence spectrum

$$P_\kappa(\ell) = \frac{9H_0^4\Omega_{m,0}^2}{4}\int_0^{\chi_s}\left(\frac{\chi_s - \chi}{\chi_s a}\frac{F_0}{F}\right)^2 P_{\delta_m}\left[\frac{\ell}{\chi}, \chi\right]d\chi . \quad (38)$$

Let us consider the action (2) with the potential (5). In the deep matter era where the Ricci scalar R is much larger than H_0^2, we have $M^2/F \gg k^2/a^2$ and $F \simeq 1$ for the wavenumber k relevant to the matter power spectrum

[22]. Since $G_{\rm eff} \simeq G$ in this regime from Eq. (24), the perturbations evolve in a standard way: $\delta_m \propto t^{2/3}$ and $\Phi_{\rm wl} = {\rm constant}$. Meanwhile, at the late epoch of the matter era, the system can enter a stage characterized by the condition $M^2/F \ll k^2/a^2$. Since $G_{\rm eff} \simeq (1 + 2Q^2)/8\pi F$ during this stage, the perturbations evolve in a non-standard way:

$$\delta_m \propto t^{(\sqrt{25+48Q^2}-1)/6}, \quad \Phi_{\rm wl} \propto t^{(\sqrt{25+48Q^2}-5)/6}. \quad (39)$$

The critical redshift z_k at $M^2/F = k^2/a^2$ can be estimated as

$$z_k \simeq \left[\left(\frac{k^2}{H_0^2} \frac{1}{Q^2(1-p)} \right)^{1-p} \frac{2^p p C}{(3F_0 \Omega_{m,0})^{2-p}} \frac{V_1}{H_0^2} \right]^{\frac{1}{4-p}} - 1. \quad (40)$$

As long as $z_k \gtrsim 1$ it is expected that the effect of modified gravity manifests itself in weak lensing observations.

Since the evolution of perturbations is similar to that in the ΛCDM model at an early epoch characterized by the condition $z \gg z_k$, the deflecting potential $\Phi_{\rm wl}$ at late times is given by [43]

$$\Phi_{\rm wl}(k,a) = \frac{9}{10} \Phi_{\rm wl}(k,a_i) T(k) \frac{D(k,a)}{a}, \quad (41)$$

where $\Phi_{\rm wl}(k,a_i) \simeq 2\Phi(k,a_i)$ corresponds to the initial deflecting potential generated during inflation, $T(k)$ is a transfer function that describes the epochs of horizon crossing and radiation/matter transition ($50 \lesssim z \lesssim 10^6$), and $D(k,a)$ is the growth function at late times defined by $D(k,a)/a = \Phi_{\rm wl}(a)/\Phi_{\rm wl}(a_I)$ (a_I corresponds to the scale factor at a redshift $1 \ll z_I < 50$).

Since we are interested in the case where the transition redshift z_k is smaller than 50, we can use the standard transfer function of Bardeen *et al.* [44]:

$$T(x) = \frac{\ln(1 + 0.171x)}{0.171x} \Big[1.0 + 0.284x + (1.18x)^2$$
$$+ (0.399x)^3 + (0.490x)^4 \Big]^{-0.25}, \quad (42)$$

where $x \equiv k/k_{\rm EQ}$ and $k_{\rm EQ} = 0.073\, \Omega_{m,0} h^2\, {\rm Mpc}^{-1}$.

In the ΛCDM model the growth function during the matter-dominated epoch ($\Omega_m = 1$) is scale-independent: $D(k,a) = a$.[1] In our scalar-tensor model the mass squared M^2 given in Eq. (21) evolves as $M^2 \propto t^{-2(2-p)/(1-p)}$, which implies that the transition time t_k at $M^2/F = k^2/a^2$ has a scale-dependence $t_k \propto k^{-\frac{3(1-p)}{4-p}}$ [22]. This leads to the scale-dependent growth of metric perturbations.

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[1] Note that in the late-time accelerated epoch the growth of matter pertubations is no longer described by $D(a) = a$.

Using Eqs. (29) and (41) we obtain the matter perturbation δ_m at the redshift $z < z_I$:

$$\delta_m(k,a) = -\frac{3}{10} \frac{F}{F_0} \frac{k^2}{\Omega_{m,0} H_0^2} \Phi_{\rm wl}(k,a_i) T(k) D(k,a). \quad (43)$$

The initial power spectrum generated during inflation is $P_{\Phi_{\rm wl}} \equiv 4|\Phi|^2 = (200\pi^2/9k^3)(k/H_0)^{n_s-1}\delta_H^2$, where n_s is the spectral index and δ_H^2 is the amplitude of $\Phi_{\rm wl}$. Then the power spectrum, $P_{\delta_m} \equiv |\delta_m|^2$, is given by

$$P_{\delta_m}(k,a) = 2\pi^2 \left(\frac{F}{F_0} \right)^2 \frac{k^{n_s}}{\Omega_{m,0}^2 H_0^{n_s+3}} \delta_H^2 T^2(k) D^2(k,a). \quad (44)$$

From Eqs. (38) and (44) we get

$$\begin{aligned} P_\kappa(\ell) &= \frac{9\pi^2}{2} \int_0^{z_s} \left(1 - \frac{X}{X_s} \right)^2 \frac{1}{E(z)} \delta_H^2 \\ &\times \left(\frac{\ell}{X} \right)^{n_s} T^2(x) \left(\frac{\Phi_{\rm wl}(z)}{\Phi_{\rm wl}(z_I)} \right)^2 {\rm d}z, \quad (45) \end{aligned}$$

where

$$E(z) = \frac{H(z)}{H_0}, \quad X = H_0\chi, \quad x = \frac{H_0}{k_{\rm EQ}} \frac{\ell}{X}. \quad (46)$$

From Eq. (33) the quantity X satisfies the differential equation ${\rm d}X/{\rm d}z = 1/E(z)$. In the following we use the value $z_s = 1$ in our numerical simulations.

IV. OBSERVATIONAL SIGNATURES OF MODIFIED GRAVITY

When $Q \neq 0$ the evolution of δ_m during the time-interval $t_k < t < t_\Lambda$ (where t_Λ is the time at $\ddot{a} = 0$) is given by Eq. (39), whereas $\delta_m \propto t^{2/3}$ in the ΛCDM model ($Q = 0$). Hence, at time t_Λ, the power spectrum for $Q \neq 0$ exhibits a difference compared to the ΛCDM model [22]:

$$\frac{P_{\delta_m}(t_\Lambda)}{P_{\delta_m}^{\Lambda{\rm CDM}}(t_\Lambda)} = \left(\frac{t_\Lambda}{t_k} \right)^{2\left(\frac{\sqrt{25+48Q^2}-1}{6} - \frac{2}{3} \right)} \propto k^{\Delta n(t_\Lambda)}, \quad (47)$$

where

$$\Delta n(t_\Lambda) = \frac{(1-p)(\sqrt{25+48Q^2}-5)}{4-p}. \quad (48)$$

In order to derive the difference $\Delta n(t_0)$ at the present epoch, we need to solve perturbation equations numerically by the time t_0. However, as long as z_k is larger than the order of unity, the growth rate of δ_m during the time-interval $t_\Lambda < t < t_0$ hardly depends on k for fixed Q. Hence it is expected that the analytic estimation (48) does not differ much from $\Delta n(t_0)$ provided $z_k \gg 1$.

We start integrating the background equations (16)-(18) from the deep matter era and identify the present
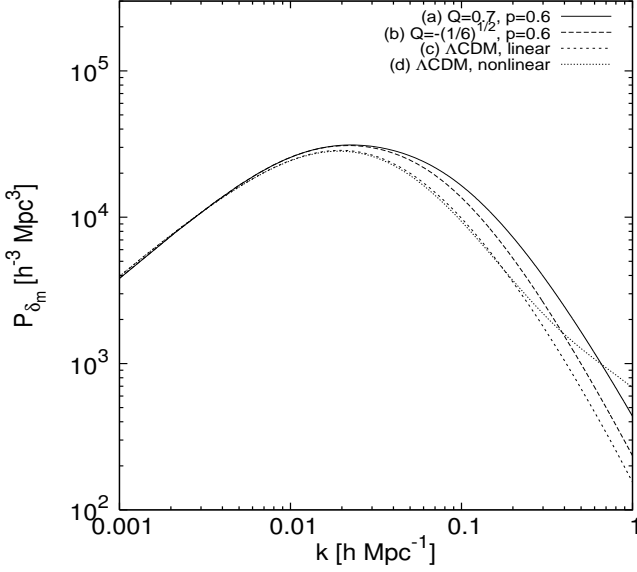
Figure 1: The matter power spectra $P_{\delta_m}(k)$ at the present epoch for (a) $Q = 0.7$, $p = 0.6$, $C = 0.9$, (b) $Q = -1/\sqrt{6}$, $p = 0.6$, $C = 0.9$, (c) the ΛCDM model, and (d) the ΛCDM model with a nonlinear halo-fitting ($\sigma_8 = 0.78$ and shape parameter $\Gamma = 0.2$). The model parameters are $\Omega_{m,0} = 0.28$, $H_0 = 3.34 \times 10^{-4} h \, \mathrm{Mpc}^{-1}$, $n_s = 1$ and $\delta_H^2 = 3.2 \times 10^{-10}$. In the cases (a) and (b) we start integrating Eqs. (16)-(18) with initial conditions $(x_1, x_2, x_3) = (0, [(3 + 2Q\lambda - 6Q^2)/2\lambda^2]^{1/2}, 0)$ and $F - 1 = -10^{-8}$.

epoch by the condition $\Omega_m = 0.28$. We then run the code again from $z = z_I \, (< 50)$ to $z = 0$ in order to solve the perturbation equations (25) and (31). Since we are considering the case in which z_k is smaller than z_I, the initial conditions for matter perturbations are chosen to be $\frac{d\delta_m}{dN} = \delta_m$ (i.e., those for the ΛCDM model).

In Fig. 1 we plot the matter power spectra at the present epoch for (a) $Q = 0.7$, $p = 0.6$, $C = 0.9$, (b) $Q = -1/\sqrt{6}$, $p = 0.6$, $C = 0.9$, (c) the ΛCDM model, and (d) the ΛCDM model with a nonlinear halo-fitting [45]. Since we do not take into account nonlinear effects in the cases (a)-(c), these results are trustable in the linear regime $k \lesssim 0.2 h \, \mathrm{Mpc}^{-1}$.

In the case (b), which corresponds to $f(R)$ gravity with $n = 0.75$ in the model (3), the spectrum shows a deviation from the ΛCDM model for $k > 0.01 h \, \mathrm{Mpc}^{-1}$. On the scales $k = 0.01 h \, \mathrm{Mpc}^{-1}$ and $k = 0.1 h \, \mathrm{Mpc}^{-1}$ the critical redshifts at $M^2/F = k^2/a^2$ are given by $z_k = 2.995$ and $z_k = 5.868$, respectively. Numerically we find $\Delta n(t_0) = 0.017$ and $\Delta n(t_0) = 0.119$ for $k = 0.01 h \, \mathrm{Mpc}^{-1}$ and $k = 0.1 h \, \mathrm{Mpc}^{-1}$ respectively, whereas the estimation (48) gives the value $\Delta n(t_\Lambda) = 0.088$. Since z_k decreases for smaller k, the analytic estimation (39) obtained by using the condition $z_k \gg 1$ tends to be invalid on larger scales. This is the main reason of the discrepancy between $\Delta n(t_0)$ and $\Delta n(t_\Lambda)$ found for $k < 0.1 h \, \mathrm{Mpc}^{-1}$. We checked that $\Delta n(t_0)$ approaches the analytic value $\Delta n(t_\Lambda) = 0.088$ on smaller scales, e.g., $\Delta n(t_0) = 0.089$
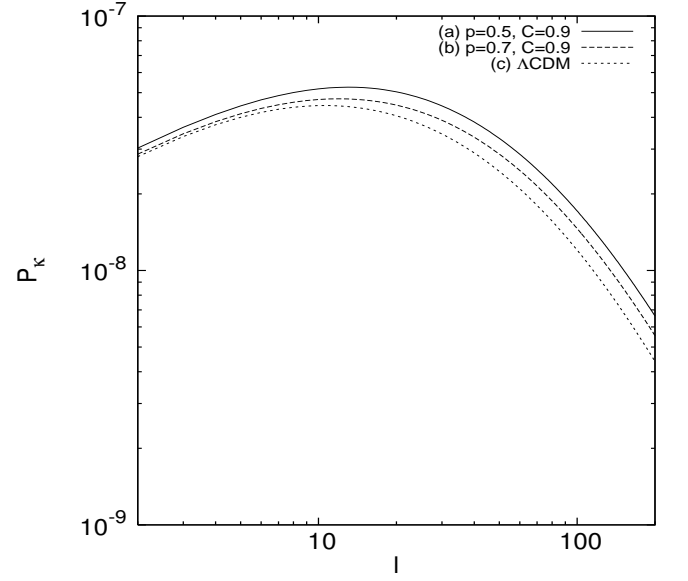

Figure 2: The convergence power spectrum $P_\kappa(\ell)$ in $f(R)$ gravity ($Q = -1/\sqrt{6}$) for the cases: (a) $p = 0.5$, $C = 0.9$ and (b) $p = 0.7$, $C = 0.9$. We also show the spectrum in the ΛCDM model. Other model parameters are chosen similarly as in the case of Fig. 1.

for $k = 4.3 h \, \mathrm{Mpc}^{-1}$.

For larger $|Q|$ the growth rate of δ_m increases in the regime $z_\Lambda < z < z_k$, which alters the shape of the matter power spectrum. In the case (a) of Fig. 1 we numerically find that $\Delta n(t_0) = 0.323$ on the scale $k = 0.1 h \, \mathrm{Mpc}^{-1}$, while the estimation (48) gives $\Delta n(t_\Lambda) = 0.231$. Again this analytic estimation is in a better agreement with $\Delta n(t_0)$ on smaller scales, e.g., $\Delta n(t_0) = 0.244$ for $k = 4.3 h \, \mathrm{Mpc}^{-1}$. In Fig. 1 we also show the matter power spectrum in the ΛCDM model derived by using the nonlinear halo-fit [45]. This gives rise to an enhancement of the power in the nonlinear regime ($k > 0.2 h \, \mathrm{Mpc}^{-1}$). The spectrum in the case (a) exhibits a significant difference compared to this halo-fit ΛCDM spectrum even for $k < 0.2 h \, \mathrm{Mpc}^{-1}$, which implies that our linear analysis is enough to place stringent constraints on model parameters Q and p from observations of galaxy clustering.

Let us next proceed to the convergence power spectrum of weak lensing. Compared to the matter power spectrum the wavenumber k is replaced by $k = \ell/\chi$. In the deep matter era the evolution of the Hubble parameter can be approximated as $H^2(z) \simeq H_0^2 \Omega_{m,0}(1+z)^3$, which gives $\chi \simeq 2/(H_0 \Omega_{m,0}^{1/2}) = \mathrm{constant}$. Hence the time t_ℓ at $M^2/F = (\ell/\chi)^2/a^2$ has an ℓ-dependence $t_\ell \propto \ell^{-\frac{3(1-p)}{4-p}}$, provided this transition occurs at the redshift $z_\ell \gg 1$.

Since $\Phi_{\mathrm{wl}} \simeq \mathrm{constant}$ for $t_I < t < t_\ell$ and $\Phi_{\mathrm{wl}} \propto t^{(\sqrt{25+48Q^2}-5)/6}$ for $t_\ell < t < t_\Lambda$, we have that

$$\frac{\Phi_{\mathrm{wl}}(z_\Lambda)}{\Phi_{\mathrm{wl}}(z_I)} \simeq \left(\frac{t_\Lambda}{t_\ell}\right)^{(\sqrt{25+48Q^2}-5)/6} . \quad (49)$$
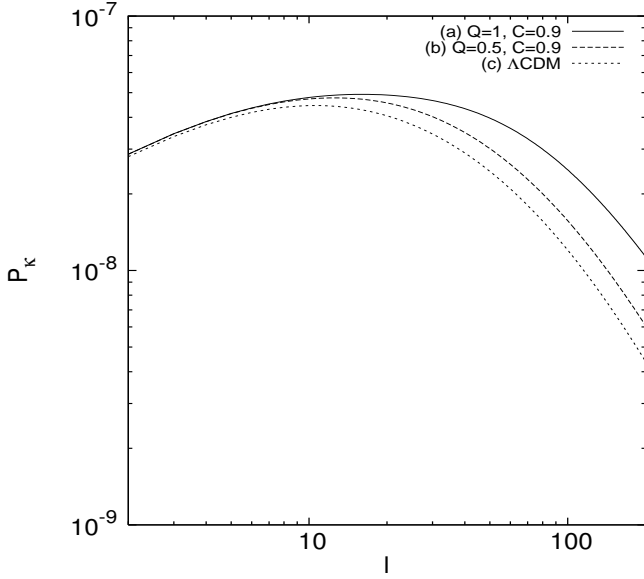
Figure 3: The convergence power spectrum $P_\kappa(\ell)$ for $p = 0.7$ with two cases: (a) $Q = 1$, $C = 0.9$ and (b) $Q = 0.5$, $C = 0.9$ together with the ΛCDM spectrum. Other model parameters are chosen similarly as in the case of Fig. 1.

As long as $z_\ell \gg 1$, the evolution of $\Phi_{\rm wl}$ during the time-interval $t_\Lambda < t < t_0$ is almost independent of ℓ for a fixed value of Q. Then we obtain the following ℓ-dependence for $0 < z < z_\Lambda \sim z_s$:

$$\left(\frac{\Phi_{\rm wl}(z)}{\Phi_{\rm wl}(z_I)}\right)^2 \propto \ell^{\frac{(1-p)(\sqrt{25+48Q^2}-5)}{4-p}} . \tag{50}$$

From Eq. (45) this leads to a difference of the spectral index of the convergence spectrum compared to the ΛCDM model:

$$\frac{P_\kappa(\ell)}{P_\kappa^{\Lambda\rm CDM}(\ell)} \propto \ell^{\Delta n} , \tag{51}$$

where Δn is the same as $\Delta n(t_\Lambda)$ given in Eq. (48). We caution again that the estimation (51) is valid for $z_\ell \gg 1$.

In Fig. 2 we plot the convergence spectrum in $f(R)$ gravity for two different values of p together with the ΛCDM spectrum. We focus on the linear regime characterized by $\ell \lesssim 200$. Since the ΛCDM model corresponds to the limit $n \to \infty$ in Eq. (3), the power $p = 2n/(2n+1)$ approaches 1 in this limit. The deviation from the ΛCDM model becomes important for smaller p away from 1.

When $p = 0.7$, for example, Fig. 2 shows that such a deviation becomes significant for $\ell \gtrsim 10$. Numerically we get $\Delta n = 0.056$ at $\ell = 200$, which is slightly smaller than the analytic value $\Delta n = 0.068$ estimated by Eq. (51). The main reason for this difference is that the critical redshift $z_\ell = 3.258$ at $\ell = 200$ is not very much larger than unity.

When $p = 0.5$ the deflecting potential $\Phi_{\rm wl}$ is amplified even for small ℓ ($\lesssim 10$), which is associated with the fact that z_ℓ is greater than 1 even for $\ell > 2$. For example we find that $z_\ell = 1.386$ for $\ell = 5$. In this case the system enters the non-standard regime ($z < z_\ell$) before entering the epoch of an accelerated expansion ($z < z_\Lambda \sim 1$), which leads to the amplification of $\Phi_{\rm wl}$. This changes the total amplitude of $P_\kappa(\ell)$ relative to the ΛCDM model. The numerical value of Δn at $\ell = 200$ is found to be $\Delta n = 0.084$ for $p = 0.5$. Since Δn increases for smaller p, this information is useful to place a lower bound on p in $f(R)$ gravity from weak lensing observations.

In Fig. 3 the convergence spectrum for $p = 0.7$ is plotted for two different values of Q together with the ΛCDM spectrum. We note that the transition redshift z_ℓ decreases for larger $|Q|$, see Eq. (40). Hence the deviation from the ΛCDM model is insignificant for small ℓ, unless we choose smaller values of p. However the spectrum is strongly modified for $\ell \gtrsim 10$ with the increase of $|Q|$. The numerical values of Δn at $\ell = 200$ are found to be $\Delta n = 0.084$ and $\Delta n = 0.311$ for $Q = 0.5$ and $Q = 1$, respectively. Hence it should be possible to derive an upper bound on the strength of the coupling Q by using observational data of weak lensing.

V. CONCLUSIONS

We have discussed the signature of modified gravity in weak lensing observations. Our model is described by the action (2) with a constant coupling Q, which is equivalent to Brans-Dicke theory with a field potential V. This theory includes $f(R)$ gravity as a special case ($Q = -1/\sqrt{6}$). The scalar-field potential $V(\phi)$ can be designed to satisfy local gravity constraints through a chameleon mechanism. The representative potential that satisfies LGC is given in Eq. (5), which is motivated by viable $f(R)$ models proposed by Hu and Sawicki [15] and by Starobinsky [16]. Note that most of past works in scalar-tensor dark energy models restricted the analysis in the small coupling region ($|Q| \lesssim 10^{-3}$). In this paper we focused on the large $|Q|$ region in which a significant difference from the ΛCDM model can be expected in weak lensing observations.

Cosmologically these models can show deviations from the ΛCDM model at late epochs of the matter-dominated era. The growth rate of matter density perturbations gets larger for redshifts smaller than a critical value z_k. Since z_k increases for larger k, the matter power spectrum is subject to change on smaller scales. We evaluated the matter power spectrum $P_{\delta_m}(k)$ numerically and showed that the spectral index and the amplitude of $P_{\delta_m}(k)$ can be significantly modified for larger values of $|Q|$.

The non-standard evolution of matter perturbations affects the convergence power spectrum $P_\kappa(\ell)$ of weak lensing. As long as the transition redshift z_ℓ is larger than the order of unity, one can estimate the difference Δn of spectral indices between modified gravity and the ΛCDM cosmology to be $\Delta n \simeq (1-p)(\sqrt{25 + 48Q^2} - 5)/(4-p)$ with $0 < p < 1$. In $f(R)$ gravity the parameter n for the model (3) is linked with the parameter p via

the relation $p = 2n/(2n + 1)$. The limit $p \to 1$ (i.e., $n \to \infty$) corresponds to the ΛCDM model, in which case we have $\Delta n \to 0$. The difference of the convergence spectrum relative to the ΛCDM case is significant for p away from 1. As seen in Fig. 2 (which corresponds to the case $Q = -1/\sqrt{6}$), the spectral index and the amplitude of $P_\kappa(\ell)$ are modified for smaller values of p.

If we take larger values of $|Q|$, the convergence spectrum deviates from that in the ΛCDM model more significantly. This situation is clearly seen in the numerical simulation of Fig. 3. It should be possible to place strong observational constraints on the parameters Q and p by using observational data of weak lensing and the matter power spectrum, which we leave for future work. We hope that some signatures of modified gravity can be detected in future high-precision observations to reveal the origin of dark energy.

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